

Problem : Hiding Hazelnuts - Solution

First let us take a look at when a distribution is stable. This is the case when none of the cabinets are "too full", i.e. when all heights are smaller or equal then 2 in the configuration.

Now notice that from any stable configuration, we can get to the configuration in which all cabinets contain two hazelnuts, i.e. this configuration will occur infinitely often and is therefore recurrent. This configuration is stable as well.

Therefore to find all stable and recurrent configurations, we want to find out which stable configurations we can reach when we start from this configurations where there are two hazelnuts in every cabinet.

Now let us take a deeper dive into stable configurations that are not recurrent. Note that any configuration in which two adjacent cabinets contain only one nut are not recurrent, as once at least one of the two cabinets contains an extra nut, they will never contain only 1 at the same time again. This is because the only way to get only one hazelnut in a cabinet is if there are just some distributed from the cabinet to its neighbours, which by definition means that the neighbour does not contain just one hazelnut.

Now let us look at what happens when adding some particle to a site of height 2 in some configuration. For example take this configuration:

22121222 $\dot{1}$ 222,

where we add an extra hazelnut to the dotted site. We then get the following configuration:

221212231222,

which when we stabilize it looks like this:

221221222222.

First observe that left of the 1 that is left of the dotted site and right of the 1 that is right of the dotted site nothing happened. This is not a fortunate example if one thinks about it. In fact, what happens is when we add a particle of a site of height 2, that the first 1 left of the site and the first 1 right of the site will end up being a 2, there will be one site in between that will be 1 and the rest will be 2 as well. The site that in the end has height 1 can be determined in the following way: put a mirror in the middle of the left and the right 1. The site in the mirror image of the dotted sight will be the site that ends after stabilization with height 1.

After trying some more examples where there is no left or right 1 from the dotted site, we see that then the end or beginning respectively acts as this 1.

Using this logic, one can argue that from the configuration with height 2 at every site by adding (multiple) particles can only result in a configuration with at most 1 site with height 1.

In conclusion we find that all recurrent and stable configurations are the ones where all sites have height 2 with at most one site with height 1.